Sensors – Circuits Overview

Electrical Circuits:

Electrical devices and systems are used for communications, information processing, and power. The physical scale range from millions of computing elements on a square centimeter of semiconductor chip to electrical power grids that span the continent. These applications are based on the movement and control of electrons. Figure 1 shows symbols for common electrical devices or elements. Passive elements dissipate or store energy. Active elements such as sources can supply energy. Conductors, which provide little resistance to electron flow, are used for routing. Insulators, which offer great resistance to electron flow, isolate devices and define flow paths. The spatial resistivity of semiconductors can be manipulated to create complex electrical behavior. Other devices can convert electrical energy to mechanical energy or vice versa. Optoelectronic devices combine optical and electronic functions. These devices are combined into circuit networks. The term “circuit” emphasizes the need for closed current paths in a network. Electrical characterization is done in terms of current, voltage, and power.

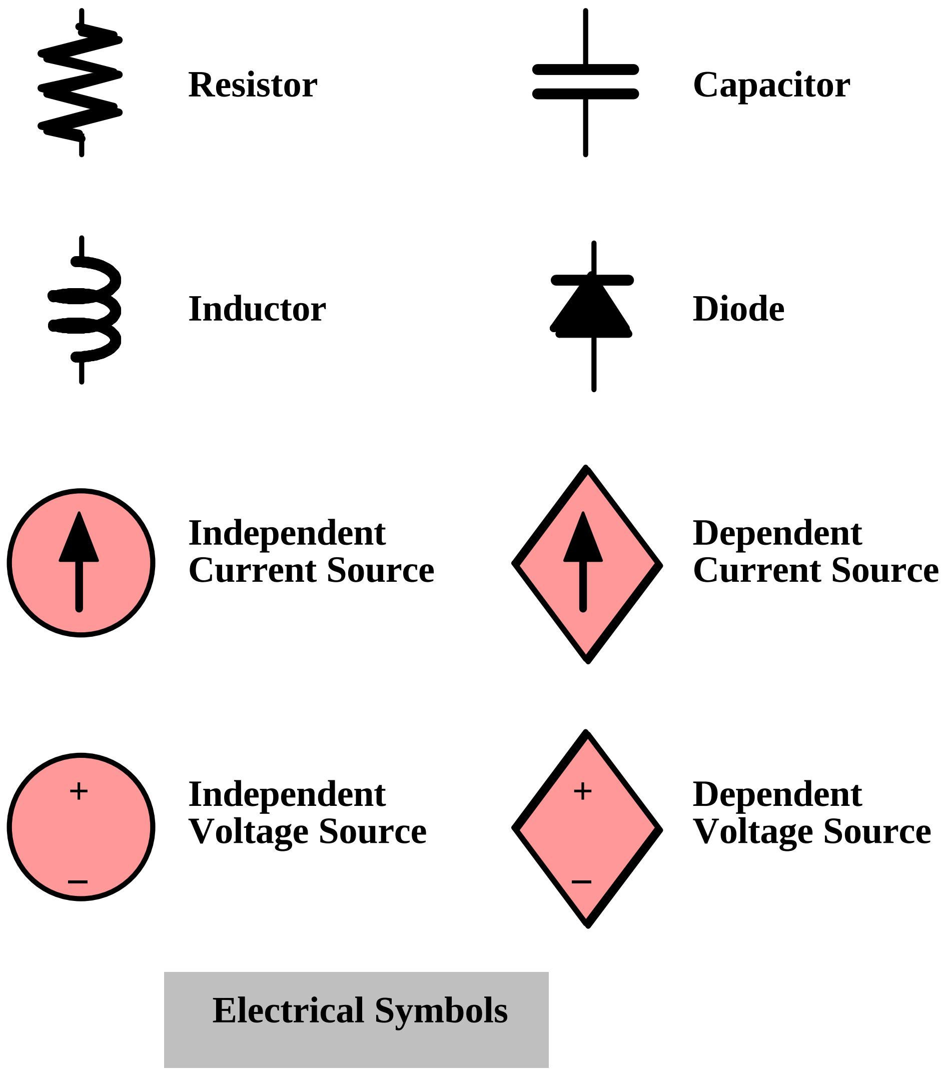

Figure 1.

A circuit model represents a physical network or a device as a connected collection of ideal elements. The behavior of the model is approximately the same as the actual circuit within some range of conditions. The circuit operating conditions are often limited to DC or AC. DC stands for direct current and indicates that all voltages and currents are constant. AC stands for alternating current and indicates that all currents and voltages are sinusoidal functions of time. The ideal elements may include passive elements, active sources, and other specialized devices. Passive elements are resistors, inductors, capacitors, and diodes. Independent sources supply the current needed to maintain a constant voltage or prescribed voltage function or supply the voltage needed to maintain a constant current or prescribe current function. Dependent sources maintain a voltage or current which depends on a voltage or a current elsewhere in the network. Specialized devices include diodes which permit current flow in one direction only or transistors which control a voltage or a current with another voltage or current.

Figure 2 shows a two-terminal electrical device with the standard sign convention. The electrical quantities of current, voltage, and power are represented by symbols i, v, and p, respectively. For instantaneous values, the symbols are lower case, as shown, and the quantities are generally functions of time. For average values or DC conditions, the symbols become I, V, and P and the quantities are constants. Note that the average value for periodic functions are calculated over a period or multiple periods. Direction of currents and polarity of voltages are usually indicated by two subscripts indicating points in the circuit network. Positive currents flow from the first point listed to the second. The plus point for voltages is listed first and the negative point second. Power for devices is indicated by a single subscript matching the corresponding device label.

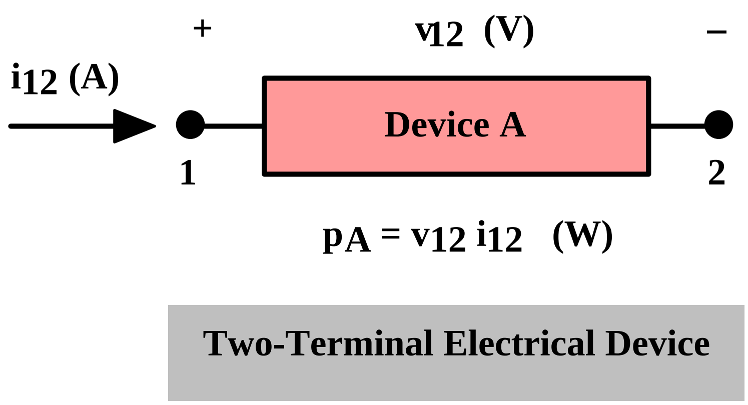

Figure 2.

Current – Current is a base quantity in the SI system of units. It is the amount of charge that moves across a surface per unit time. The unit is amperes with symbol A. (Note that charge is defined in terms of current. One coulomb of charge is produced by one ampere of charge flowing through a surface in one second.) Consequently, it has a prescribed magnitude and direction. A positive current represents a net flow of positive charge going in the prescribed direction or a net flow of negative charge going in the opposite direction. For electrical systems, the physical case is the latter. Electrons are the mobile atomic particle and have a negative charge of $- q = -1.602 \times 10^{-19}$ C. This sometimes confusing sign situation is a result of historical convention. Note that small currents in the milliampere range involve the movement of a massive number of electrons.

Voltage – Voltage is a measure of electrical potential energy between two points. It is the change in energy, i.e. work, required to move a charge between the two points. The unit is volts with symbol V. (Note that one volt is equivalent to one joule of energy per coulomb of charge.) Voltage has a magnitude and orientation. A positive voltage represents the potential energy of a charge at the plus point with respect to the negative point. Positive current flows from the positive voltage point to the negative voltage point. Current does not need to flow between the points for the voltage to exist. The situation is analogous to a mass in a gravity field. The mass on a shelf has gravitational potential energy with respect to the floor. This energy is dissipated if the mass falls to the floor. Representations of electrical systems often have a "ground" identified. All voltages are stated with respect to this point. Otherwise voltages between points must be explicitly shown.

Power – Electrical power is a measure of energy absorbed per unit time by an element or system. The unit is watts with symbol W. It has a signed magnitude. Positive power means that energy was absorbed; negative power means that energy was supplied. (This is known as the passive sign convention.) Power is determined by the product of voltage times current. Positive power is indicated when the current direction is from the plus point for the voltage to the negative point. Otherwise, negative power is indicated. In order to satisfy energy conservation, all (absorbed) power in an electrical network must be zero.

Passive Electrical Parameters

Resistance – The current-voltage (IV) characteristic of resistor is shown in Figure 3. The voltage is directly proportional to the current. The slope of the IV characteristic is the resistance R. The relationship v(t) = i(t)R is known as Ohm's law. The unit is ohms with symbol . (Note that one ohm is equivalent to one volt per ampere.) The resistance of a conducting structure depends on its geometry of the structure and the resistivity of the material. The resistance is linear if R is a constant; it is nonlinear if R varies with current and voltage. The absorbed power is always positive or zero; the resistor dissipates energy converting it to heat. Note that the power can be expressed in terms of the resistance as

$$p(t) = v^2(t)/R = i^2(t)R.$$

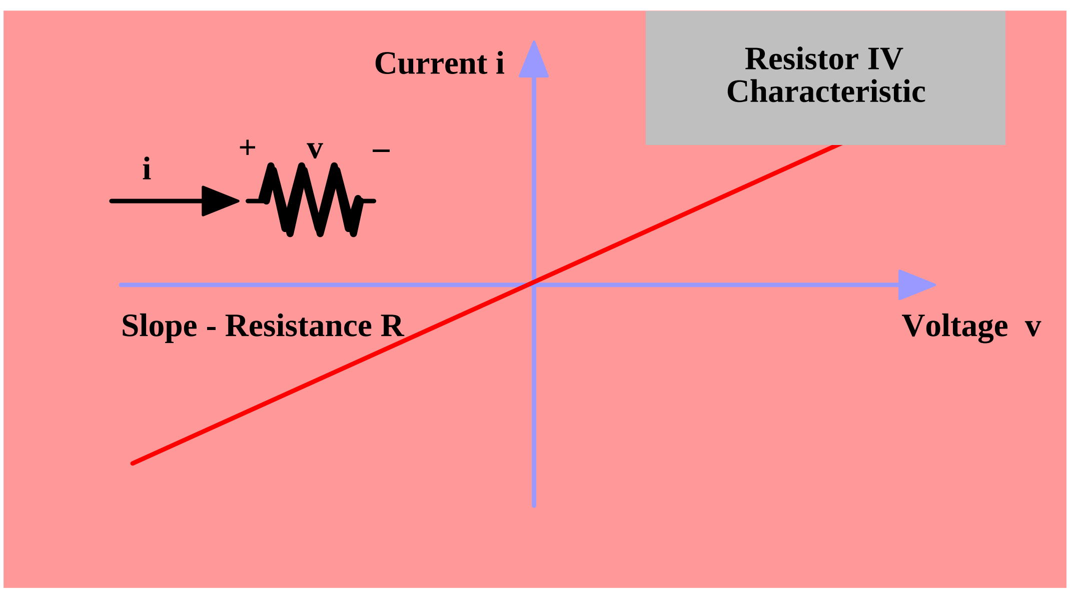

Figure 3.

Inductance – Energy may be stored in the magnetic field that exists around moving charges. If a current flows through a conductor, a magnetic field is created around the current path. Conversely, a magnetic field can cause a force on moving charge. Self-inductance L is the effect a current has on itself due to coupling of the magnetic field with the current path. Consequently, it is strongly dependent on geometry. A coil as shown in Figure 4 provides strong coupling and a large inductance. Also, a magnetic material such as iron and nickel will concentrate the magnetic field and permit more energy to be stored in the field around a current than for a nonmagnetic material. Self-inductance, or simply inductance, L is the ratio of voltage to temporal change in current, i.e.

$$v(t) = L\, di(t)/dt.$$

The unit is henry with symbol H. (Note that one henry is equivalent to one volt-second per ampere.) An inductor will store energy if the current is increasing or supply energy if the current is decreasing. The stored energy is $(1/2)Li^2(t)$.

Mutual inductance M is the effect a current has on a current in another circuit or another part of the circuit. It is the ratio of induced voltage in the secondary circuit due to a change in current in the primary circuit, i.e.

$$v(t) = M\, di(t)/dt.$$

A transformer is a device that intentionally couples the magnetic field between two coils. It can be used to convert one signal level to another, e.g. a large voltage and small current can be converted to a small voltage and a large current.

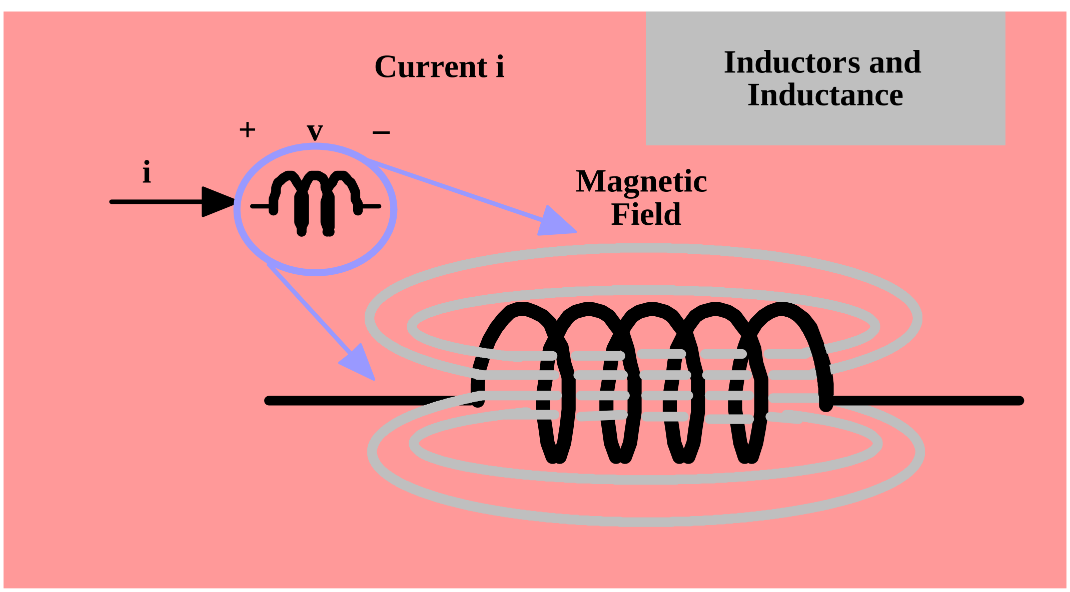

Figure 4.

Capacitance – Energy may be stored in the electric field that exists between charges. If a voltage is applied to two electrically isolated conductors in proximity, charge of opposite sign accumulates on the conductors (see Figure 5). The arrangement of the conductors and the material between the conductors determine the amount of accumulated charge. In particular, a dielectric material placed between the conductors will allow more energy to be stored than that stored for conductors separated by vacuum or air. Capacitance C is the ratio of charge Q to voltage and equivalently the ratio of current to temporal change in voltage, i.e.

$$Q = Cv(t) \quad \text{and} \quad i(t) = C\, dv(t)/dt.$$

The unit is farad with symbol F. (Note that one farad is equivalent to one coulomb per volt.) A capacitor will store energy if the voltage is increasing or supply energy if the voltage is decreasing. The stored energy is $(1/2)Cv^2(t)$.

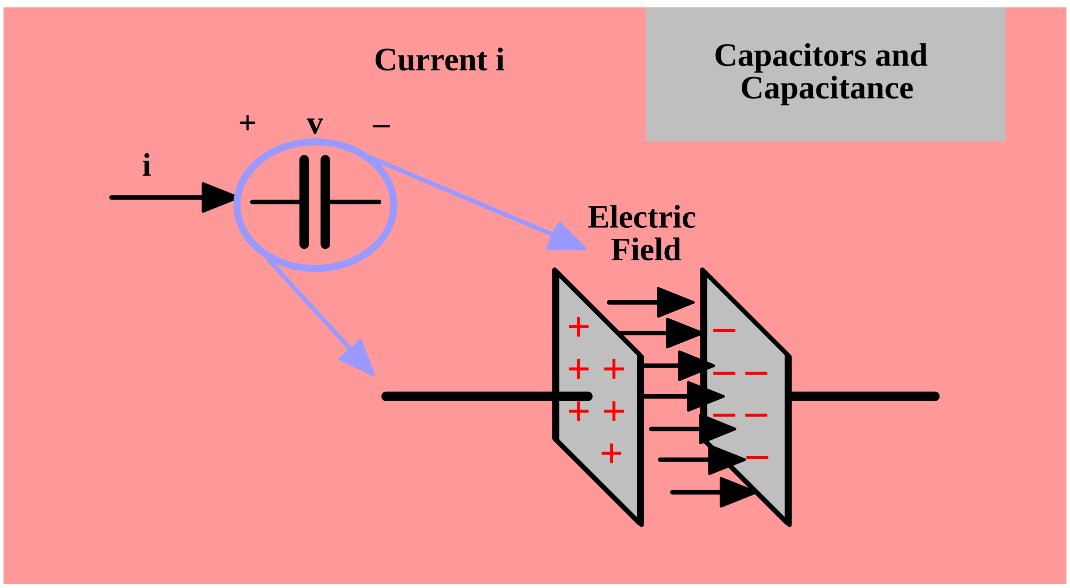

Figure 5.

Impedance –The current-voltage characteristics for resistance, inductance, and capacitance take a simple form for AC circuit operation. Impedance is relation between a sinusoidal voltage across and the sinusoidal current through a two terminal device or circuit containing several devices. The table shows the impedance relations for resistor, inductor, and capacitor networks. For a resistor network, the voltage is in phase with the current and the voltage amplitude equals the current amplitude times the resistance. Hence, the resistive impedance is frequency independent. For an inductor network, the voltage phase leads the current phase by /2 radians and the voltage amplitude equals the current amplitude times 2 times the frequency times the equivalent inductance. Hence, the inductive impedance increases for increasing frequency. For a capacitive network, the voltage phase lags the current phase by /2 radians and the voltage amplitude equals the current amplitude divided by 2 times the frequency times the equivalent capacitance.

Hence, the capacitive impedance decreases for increasing frequency. For a general network including all three elements, the phase difference and the ratio of amplitudes will be frequency dependent.

Type	Phase (v(t) verses i(t))	Amplitude Ratio (V divided by I)
Resistor	0 radians	R
Inductor	/2 radians	2 fL
Capacitor	- /2 radians	1/(2 fC)
General	varies	varies

Diodes – A diode is a nonlinear device with a current-voltage (IV) characteristic as shown in Figure 6. For negative voltages, a diode limits the current to a very small value. For positive voltages, a diode allows a large current and approaches an asymptotic voltage. This asymptotic voltage for positive currents depends on the semiconductor material used to construct the device. The turn-on voltage is the voltage for which the current becomes significant. For silicon diodes, this turn-on voltage is about 0.7 V.

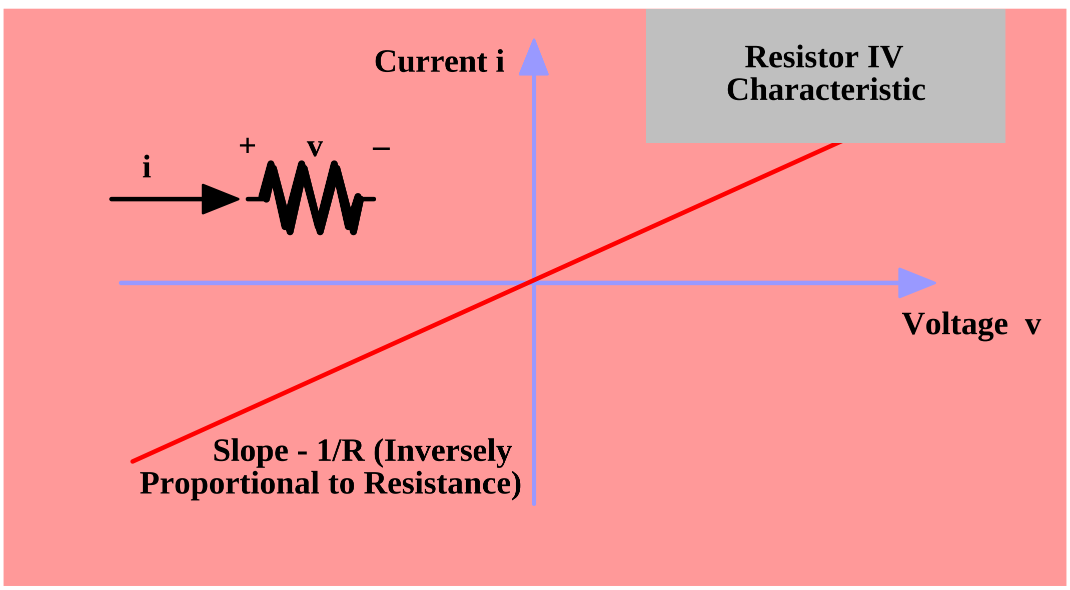

Figure 6.

Circuits Analysis:

A circuit network contains a variety of devices connected in a particular arrangement. A voltage or current in the circuit is identified as the input quantity and another voltage or current is identified as the output quantity. The behavior is typically specified in terms of a functional relationship between the input and the output quantities. The devices may have two terminals like the passive elements or multiple terminals. Multiple terminals may be connected together at nodes. Terminals are connected and current may flow if a (zero resistance) line is drawn between them. Terminals are open and current cannot flow if no line connects the terminals. Three principles are used to solve the circuit so that the output is known in terms of input and the various circuit elements – the power balance law, Kirchhoff's current law, and Kirchhoff's voltage law.

Power Balance Law – The absorbed power for all devices in a closed network must sum to zero. Conservation of energy requires that the instantaneous power absorbed by the passive elements must be supplied by the sources. Figure 7 shows a parallel set of resistors and a series set of resistors. The parallel resistors share two common nodes and thus share the same voltage. Consider the DC case. The resulting algebraic equation applying power balance and Ohm's Law can be solved as

$$0 = P_s + P_1 + P_2 + P_3 = -I_sV + V^2/R_1 + V^2/R_2 + V^2/R_3$$

or

$$I_s = V\,(1/R_1 + 1/R_2 + 1/R_3).$$

The series resistors are connected sequentially with each node having only two connections. Hence, they share a common current path, that is a common current. The resulting algebraic equation applying power balance and Ohm's Law can be solved as

$$0 = P_s + P_1 + P_2 + P_3 = -IV_s + I^2R_1 + I^2R_2 + I^2R_3)$$

or

$$V_s = I\,(R_1 + R_2 + R_3).$$

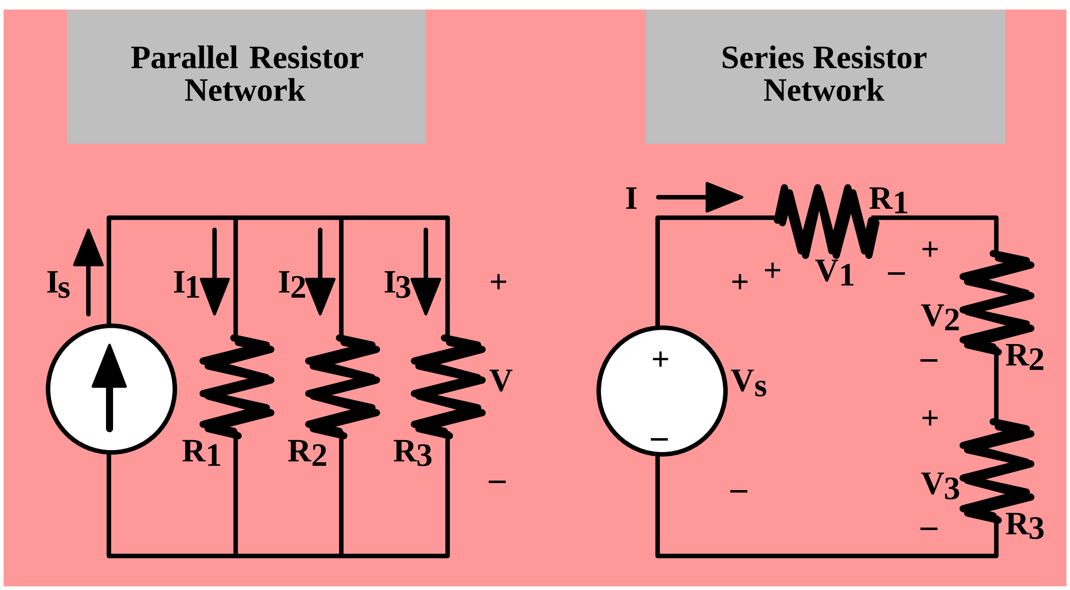

Figure 7.

Kirchhoff's Current Law – The net charge entering a closed surface in a circuit is zero. Devices, wires, etc. are normally neutral in which the negative charge of the electrons are balanced by positive charge in the atomic nuclei. Charge is not created or destroyed in any part of the network. Consequently, current going into a device, node, or wire must be balanced by current flow out. Consider the upper node in Figure 8. The current from the source enters the upper node and is I_s. Three currents leave the node through the parallel resistors: I_1, I_2, and I_3. Ohm's law gives $I_1 = V/R_1$, $I_2 = V/R_2$ and $I_3 = V/R_3$. Using the current law, the current I_s through the resistor network is

$$I_s = I_1 + I_2 + I_3 = V/R_1 + V/R_2 + V/R_3 = V\,(1/R_1 + 1/R_2 + 1/R_3).$$

The solution matches that found previously.

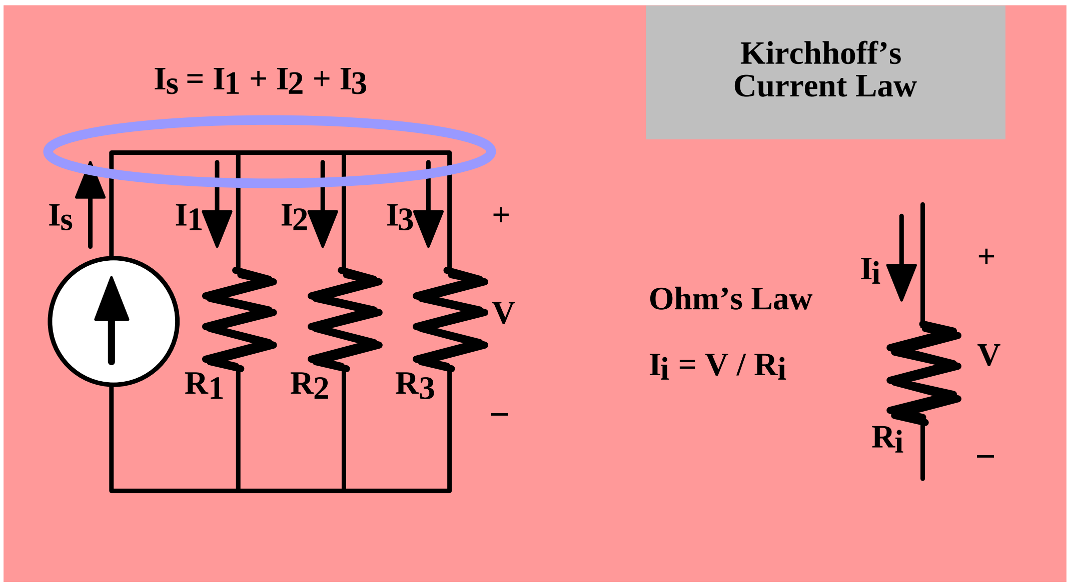

Figure 8.

Kirchhoff's Voltage Law – The potential energy associated between any two points in a circuit is independent of path. Consequently, the algebraic sum of voltages around any closed path is zero., Consider the circuit path in Figure 9. The voltage supplied by the source across the entire series combination of resistors is V_s. The three voltages across the series resistors are: V_1, V_2, and V_3. Ohm's law gives $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$. Using the voltage law, the voltage V_s across the resistor network is

$$V_s = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I\,(R_1 + R_2 + R_3).$$

The solution matches that found previously.

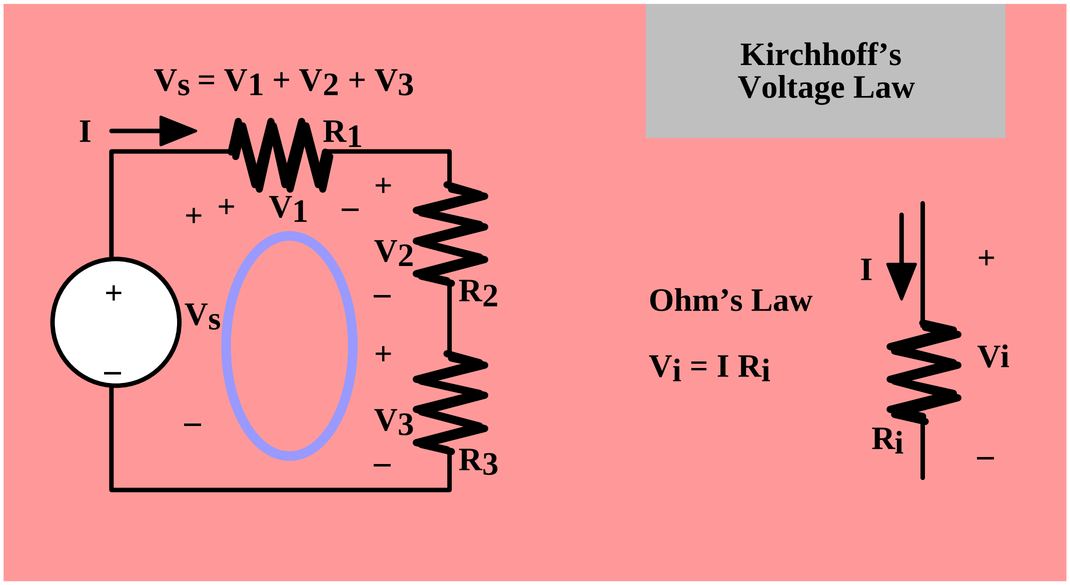

Figure 9.

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